

# FNLP - Week 6: Syntactic Parsing

Antonio León Villares

March 2022

## Contents

<b>1</b>	<b>Syntax</b>	<b>2</b>
1.1	The Need for Syntax . . . . .	2
1.2	Constituents . . . . .	2
1.3	Context Free Grammars . . . . .	4
1.4	Chomsky Normal Form . . . . .	6
<b>2</b>	<b>Syntactic Parsing</b>	<b>7</b>
2.1	The Purpose of Parsers . . . . .	7
2.2	The Issue with Structural Ambiguity . . . . .	8
2.3	Top-Down Parsing . . . . .	9
2.4	Bottom-Up Parsing . . . . .	9
2.5	The CYK Algorithm . . . . .	10
2.5.1	Worked Example: CYK . . . . .	13
<b>3</b>	<b>Statistical Parsing</b>	<b>20</b>
3.1	Probabilistic Context Free Grammars . . . . .	20
3.2	Issues with PCFGs . . . . .	22
3.3	Most Likely Parse: A Probabilistic Distribution Over Parse Trees . . . . .	23
3.4	Probabilistic CYK . . . . .	24

# 1 Syntax

## 1.1 The Need for Syntax

- **What is syntax?**
  - the way in which **words** are **arranged together**
- **Why is syntax important?**
  - necessary to develop an **accurate** model of **language**
  - BOW, N-Gram and HMMs not good enough, since they rely on a **fixed-length** history
  - for example,

*“Looking at the amazing view, he couldn’t help but gasp”*

a trigram model would have to predict “gasp”, given “help but” - unlikely for this to be successful; however, we could have easily predicted it
  - **long-range dependencies** are important for a **language model**: words depend on each other, independently of many intervening words between them:

Sam/Dogs sleeps/sleep soundly  
Sam, who is my cousin, sleeps soundly  
Dogs often stay at my house and sleep soundly  
Sam, the man with red hair who is my cousin, sleeps soundly

- **What is a theory of syntax?**
  - theory explaining which sentences are **grammatical/well formed**
  - this need not mean that a sentence is **meaningful** (i.e “Colourless green ideas sleep furiously” is grammatical, but doesn’t make sense)
  - the 2 (main) theories of syntax are:
    - \* **constituency structures**
    - \* **dependency structures**

## 1.2 Constituents

- **What is a constituent?**
  - a **group** of words (potentially a single one), which may behave as a **single unit**
  - for example, **noun phrases**
- **How can we test if a group of words is a constituent?**
  1. **Substitutability** We can “swap” constituents of the same type to produce well-formed phrases:

Dogs sleep soundly  
My next-door neighbours sleep soundly  
Green ideas sleep soundly

Figure 1: Notice, for example “My” can’t be swapped, since “My sleep soundly” doesn’t make sense.

This more generally applies to POS categories (i.e we can swap 2 adjectives, and a phrase will still make sense)

2. **Preposed/Postposed Constructions** A constituent can be placed at different places of a phrase, with the phrase still making sense:

*On September seventeenth, I'd like to fly from Atlanta to Denver.*

*I'd like to fly from Atlanta to Denver on September seventeenth.*

*I'd like to fly on September seventeenth from Atlanta to Denver .*

However, the same thing won't apply if for example we use the individual words:

*On I'd like to fly September seventeenth from Atlanta to Denver .*

3. **Coordination** We can **coordinate** constituents of the same type with conjunctions (and, or, but)

► **Pass the test:**

Her friends from Peru went to the show.

Mary and her friends from Peru went to the show.

Should I go through the tunnel?

Should I go through the tunnel and over the bridge?

► **Fail the test**

We peeled the potatoes.

\*We peeled the and washed the potatoes.

4. **Clefting** Only constituents can appear in:

\*\*\*\*\* is/are who/what/where/when/why/how ...

► **Pass the test:**

They put the boxes in the basement.

In the basement is where they put the boxes.

► **Fail the test**

They put the boxes in the basement.

\*Put the boxes is what they did in the basement.

- What is a constituent tree?

- a **tree** which breaks down a sentence into its **constituents**

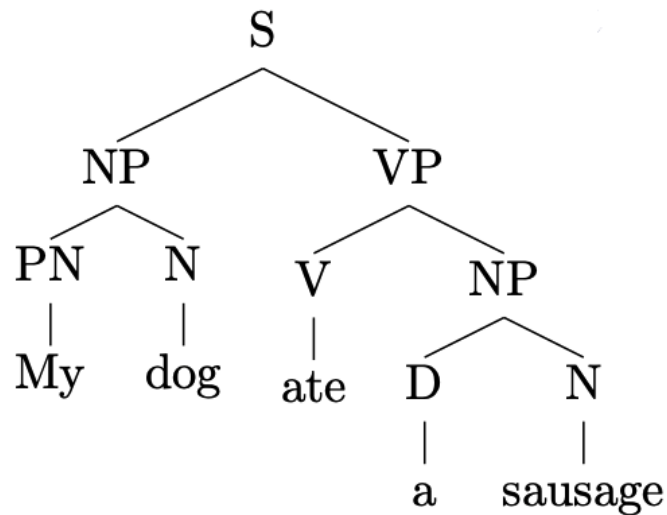


Figure 2: The internal nodes are **phrases** (i.e noun phrases like “a sandwich”), whilst the nodes immediately above words correspond to POS tags

### 1.3 Context Free Grammars

- **What is the structure of context free grammars?**

- an example of a **constituency structure**: it is built from constituents
- CFGs are (formally) a **4-tuple**:
  1. **N**: set of **non-terminal symbols** (i.e *NP* to represent a noun phrase)
  2. **Σ**: set of **terminal symbols**, disjoint from *N* (i.e words like “flight”)
  3. **R**: set of **productions** of the form:

$$A \rightarrow \beta, \quad A \in N, \beta \in \Sigma$$

4. **S**: a **start symbol**

$V = \{S, VP, NP, PP, N, V, PN, P\}$

$\Sigma = \{girl, telescope, sandwich, I, saw, ate, with, in, a, the\}$

$S = \{S\}$

$R :$

$S \rightarrow NP \ VP \quad (NP \text{ A girl}) \ (VP \text{ ate a sandwich})$

$VP \rightarrow V$

$VP \rightarrow V \ NP \quad (V \text{ ate}) \ (NP \text{ a sandwich})$

$VP \rightarrow VP \ PP \quad (VP \text{ saw a girl}) \ (PP \text{ with a telescope})$

$NP \rightarrow NP \ PP \quad (NP \text{ a girl}) \ (PP \text{ with a sandwich})$

$NP \rightarrow D \ N \quad (D \text{ a}) \ (N \text{ sandwich})$

$NP \rightarrow PN$

$PP \rightarrow P \ NP \quad (P \text{ with}) \ (NP \text{ with a sandwich})$

$N \rightarrow girl$

$N \rightarrow telescope$

$N \rightarrow sandwich$

$PN \rightarrow I$

$V \rightarrow saw$

$V \rightarrow ate$

$P \rightarrow with$

$P \rightarrow in$

$D \rightarrow a$

$D \rightarrow the$

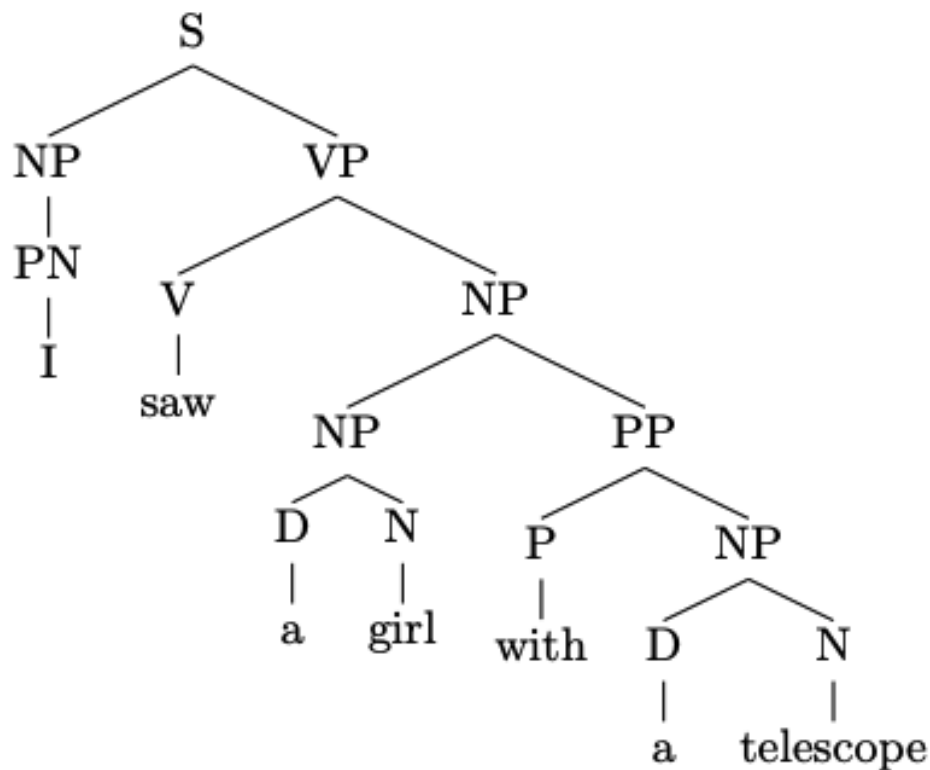


Figure 3: A possible **constituent tree** derived from the CFG above.

- **What is a derivation?**
  - a set of strings which can be produced from a CFG
  - can be represented using a parse tree
- **What are treebanks?**
  - **corpora** in which **sentences** are annotated using a **parse tree**
- **What types of equivalences can arise from different grammars?**
  - **Strong equivalence**: generate same set of strings **and** assign same phrase **structure** to each sentence
  - **Weak equivalence**: generate same set of strings (but different phrase structure assignment)
- **Why are these grammars called “context free”?**
  - the **production rules** are applicable independent of context

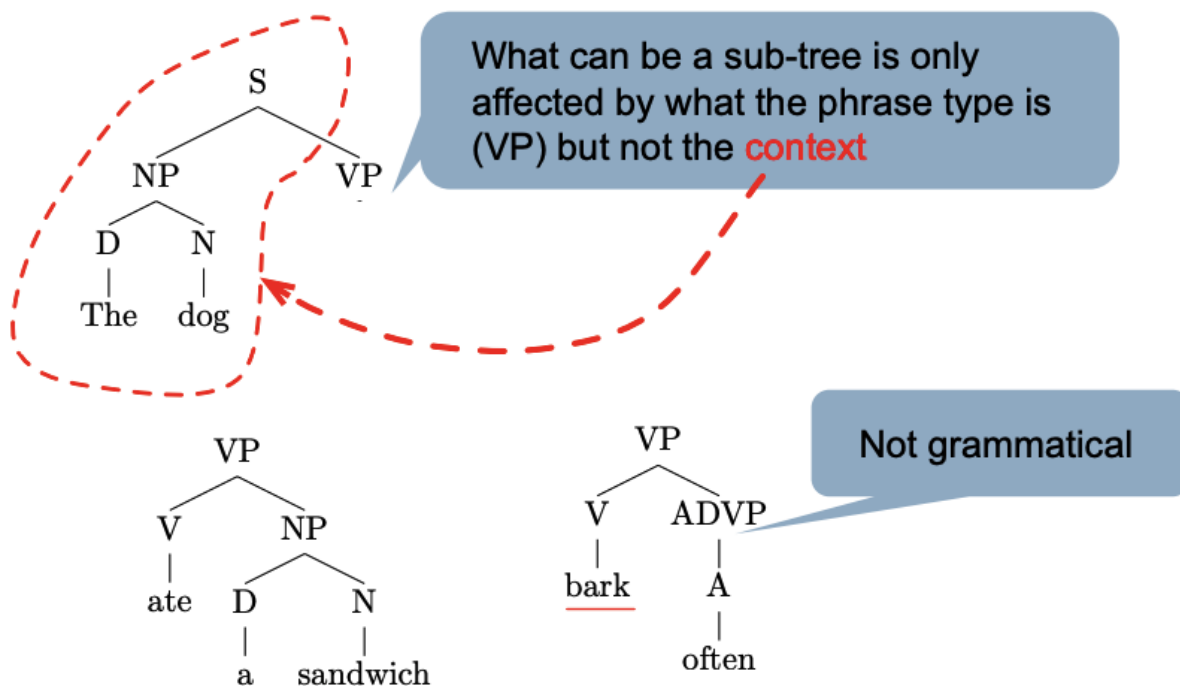


Figure 4: Here, “The dog” can be generated, without worrying about what comes after. For example, “The dog ate a sandwich” is perfectly valid.

## 1.4 Chomsky Normal Form

- **What format do grammars in Chomsky Normal Form take?**
  1. no  $\varepsilon$  productions (i.e no production rule can have the form  $A \rightarrow \varepsilon$ )
  2. a **production** can only have the following forms:

$$A \rightarrow a$$

$$A \rightarrow B C$$

where  $a$  is a **terminal**, and  $A, B, C$  are **non-terminals**

- in particular, grammars in **CNF** ensure **binary branching** (except at the terminal nodes)

- **Why are CNFs important?**

- any grammar can be **converted** to a **weakly equivalent** CNF (so same language generated, but different syntactic tree)
- CNFs are the grammars on which the **CYK Algorithm** functions

- **How can a grammar be converted to CNF?**

- rules which produce more than 2 non-terminals can be changed:

$$A \rightarrow B C D$$

gets converted to:

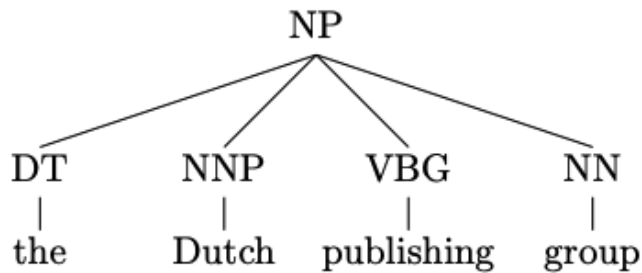
$$A \rightarrow B X \quad X \rightarrow C D$$

- **unary rules** can be converted to produce terminals:

$$C \rightarrow C_1 \implies C \rightarrow c_1$$

- you also remove  $\varepsilon$  productions (but not relevant to this course)

► **Consider**  $NP \rightarrow DT \ NNP \ VBG \ NN$



► **How do we get a set of binary rules which are equivalent?**

$$NP \rightarrow DT \ X$$

$$X \rightarrow NNP \ Y$$

$$Y \rightarrow VBG \ NN$$

► **A more systematic way to refer to new non-terminals**

$$NP \rightarrow DT \ @NP|DT$$

$$@NP|DT \rightarrow NNP \ @NP|DT\_NNP$$

$$@NP|DT\_NNP \rightarrow VBG \ NN$$

Figure 5: The format of the latter representation is useful, since it allows an easy reverse conversion for post processing.

## 2 Syntactic Parsing

### 2.1 The Purpose of Parsers

- What is syntactic parsing?

- the process of mapping a **sequence of words** to its **parse tree**
- getting this structure allows us to **interpret meaning**
- **Why is parsing important?**
  - correct structure  $\implies$  correct meaning
  - **efficiency**: impossible to search all possible structures which match a sequence of words
- **What are the 2 fundamental properties of parsers?**
  1. **Directionality**: how is the parse tree built?
    - top-down: from  $S$  to terminals
    - bottom-up: from terminals to  $S$
    - mixed: i.e start from left corner
  2. **Search Strategy**: how do we explore the space of possible parse trees, as to find the parse tree fitting our word sequence?

## 2.2 The Issue with Structural Ambiguity

- **Why is parsing hard?**
  - typical sentences can have immense **parse trees**
  - most importantly is the issue of **structural ambiguity**: how a given sequence can have **several possible parse trees**
- **How does structural ambiguity present itself?**
  1. **Attachment Ambiguity** Arises from the fact that **constituents** can be “attached” to the parse tree at different places. For example:

*“One morning I shot an elephant in my pajamas”*

represents **PP-attachment ambiguity**: we don’t know if the **prepositional phrase** “in my pajamas” attaches to:

    - “I”: the person who shot was wearing pajamas at the time of the shooting
    - “an elephant”: the elephant which got shot was wearing the shooter’s pajamas
  2. **Coordination Ambiguity** Arises from the fact that conjunctions can be applied in different ways. For example:

*“old men and women*

could refer to a group of old men, alongside women; or a group of both old men and old women.

- **Is parsing unambiguous sentences easy?**
  - sentences might be unambiguous, but still hard to parse
  - this is due to **local ambiguity**: a part of a sentence is itself ambiguous, even if the whole sentence isn’t
  - for example:

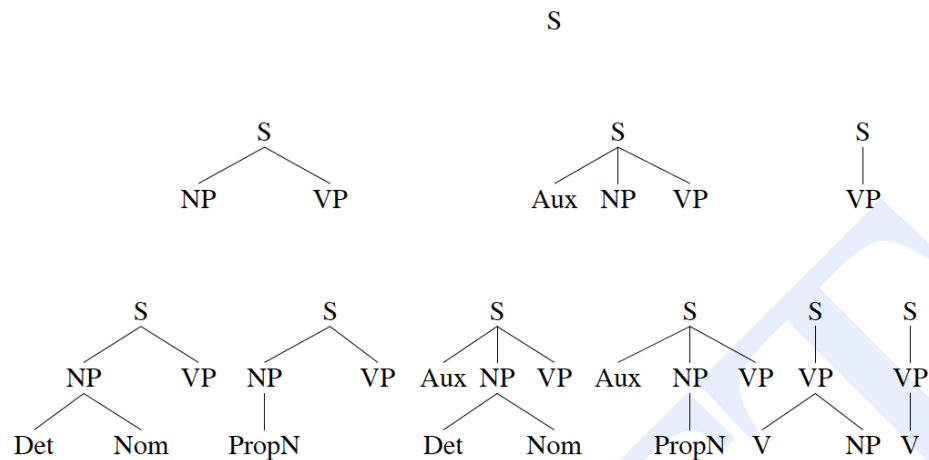
*“Book that flight”*

is unambiguous, but “book” is (it can be a verb or a noun), so a parser would have to consider both possible parsers, until it reaches the end of the parsing



## 2.3 Top-Down Parsing

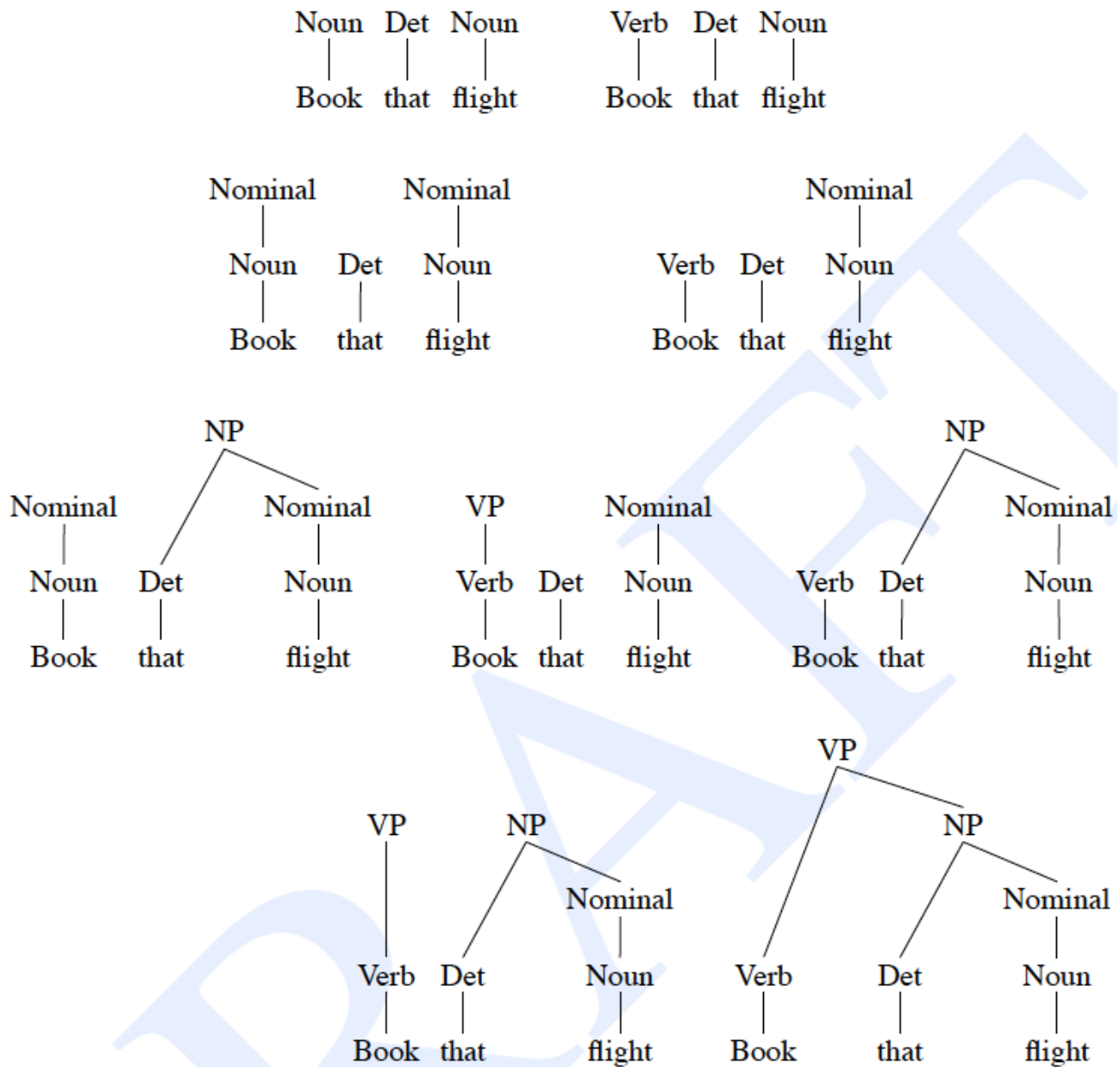
- **How does top-down parsing search the parse tree space?**
  - start with  $S$  nodes
  - choose one of the children to continue exploring
  - repeat with the node's child, until we reach a suitable parse tree
- **What is depth first search?**
  - go down a branch of the space as far as possible
  - if we reach an impossible parse tree, **backtrack**
- **What is breadth first search?**
  - expand all branches in parallel
  - generally not good: the number of branches is too large, so will take a long time to find a suitable parse tree
- **What is best first search?**
  - define a scoring function
  - score each partial parse, exploring the highest scoring option next



## 2.4 Bottom-Up Parsing

- **What is bottom-up parsing?**
  - begin with words in the sequence
  - try to build a parse tree which fits the word sequence structure
  - successful parse if  $S$  is reached

Book that flight



- How do top-down and bottom-up parsing compare?
  - top-down never explores parse trees which can't be grammatical
  - however, bottom-up explores parse trees which will always lead to parsing the sequence

## 2.5 The CYK Algorithm

- What is the CYK algorithm?
  - efficient (**dynamic programming**) bottom-up parser for CFGs
  - it applies to:
    - \* the **recognition problem** (is a sentence derived by a CFG?)
    - \* the **parsing problem** (what is the derivation tree of the sentence?)

- it relies on grammars being in CNF form
- [My IADS notes on CYK and CNF conversion](#)
- **What problems does CYK solve?**
  - **large search space:** instead of exploring the whole search space (potentially recomputing the same parse trees), it stores partial parses
  - **ambiguity:** stores **all** possible parses, so reduces the ambiguity problem
- **What is the recursive idea in CYK?**
  - the parse of a string depends on the parse of its component substrings
  - for example, to parse “Book the flight through Houston”, we need to consider whether we can parse “Book” and “the flight through Houston”
  - as a **base case**, we will be parsing individual words (i.e using POS tags)
- **What table structure does CYK employ?**
  - consider a string with  $n$  words; we store results in an  $n \times n$  table (rows from 0 to  $n - 1$ , columns from 1 to  $n$ )
  - element  $(i, j)$  contains the **partial parse** (if any) of the substring  $span(i, j)$

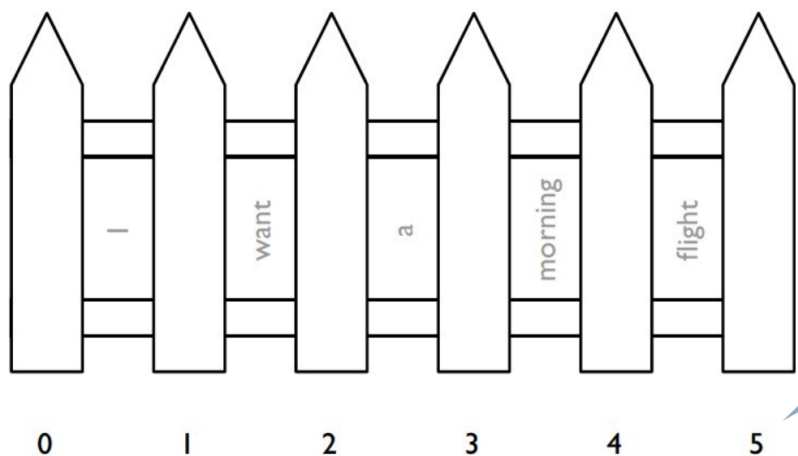


Figure 6: For example,  $span(0, 1) = \text{“I”}$ , whilst  $span(3, 5) = \text{“morning flight”}$ .

- ultimately, we want to know whether entry  $(0, n)$  contains  $S$  as a partial parse
- it must be noted that we only use the upper triangular part of the table (since we require that  $i > j$ )
- the table is filled in **top to bottom** and **left to right**, to ensure that all possible substrings are parsed beforehand
- **What is the full CYK algorithm?**
  - from the book:

```

function CKY-PARSE(words, grammar) returns table

  for  $j \leftarrow$  from 1 to LENGTH(words) do
     $table[j - 1, j] \leftarrow \{A \mid A \rightarrow words[j] \in grammar\}$ 
    for  $i \leftarrow$  from  $j - 2$  downto 0 do
      for  $k \leftarrow i + 1$  to  $j - 1$  do
         $table[i, j] \leftarrow table[i, j] \cup$ 
           $\{A \mid A \rightarrow BC \in grammar,$ 
             $B \in table[i, k],$ 
             $C \in table[k, j]\}$ 

```

Figure 7: Technically, this only recognises whether a word sequence is well-formed; to get the parse tree, we just need to ensure that each entry is paired with a **pointer**, indicating where it was derived from.

- in lectures, the table is slightly different (it is  $n \times n \times n$  and boolean, with entry  $(i, j, C)$  indicating whether  $span(i, j)$  can be parsed as  $C$  or not)

```

for each  $w_i$  from left to right

  for each preterminal rule  $C \rightarrow w_i$ 

     $chart[i - 1][i][C] = \text{true}$ 

```

Figure 8: This is how preterminal rules (i.e terminal productions) are handled.

```

for each  $max$  from 2 to  $n$ 

  for each  $min$  from  $max - 2$  down to 0

    for each syntactic category  $C$ 

      for each binary rule  $C \rightarrow C_1 C_2$ 

        for each  $mid$  from  $min + 1$  to  $max - 1$ 

          if  $chart[min][mid][C_1]$  and  $chart[mid][max][C_2]$  then

             $chart[min][max][C] = \text{true}$ 

```

Figure 9: This is how binary rules are handled. Notice, we can see the runtime will be  $\mathcal{O}(n^3|R|)$ , where  $R$  is the set of all productions in the grammar.

```

for each max from 1 to n
    for each min from max - 1 down to 0
        // First, try all binary rules as before.
        ...
        // Then, try all unary rules.
        for each syntactic category C
            for each unary rule C -> C1
                if chart[min][max][C1] then
                    chart[min][max][C] = true

```

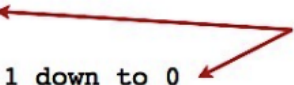


Figure 10: If we define a CNF which allows unary productions (i.e  $C_1 \rightarrow C_2$ ), we only need a slight modification.

- What does the CYK algorithm not account for?

- the algorithm can fail if there are **chains of rules**: that is, if we have  $A \rightarrow B$  or  $B \rightarrow C$ , then  $A \rightarrow B \rightarrow C \implies A \rightarrow C$  is a perfectly valid production
- however, CYK won't account for this
- the algorithm could be adjusted (i.e run repeatedly until entries don't change)
- in practice, extend the grammar to enforce **transitive closure** (i.e include a rule  $A \rightarrow C$  as part of the grammar)
- or just make sure that there are no unary rules, that is, ensure you work with a CNF, and you won't get these dumb problems

### 2.5.1 Worked Example: CYK

We consider parsing:

*“lead can poison”*

# CKY in action

lead	can	poison
0	1	2

	max = 1	max = 2	max = 3
min = 0	1 <i>N, V</i>		
min = 1		2 <i>N, M</i>	
min = 2			3 <i>N, V</i>

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

Inner rules

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Preterminal rules

Figure 11: We begin by considering the possible derivations for words. We can first think of them as POS tags.

## CKY in action

	lead	can	poison
0	1	2	3

	max = 1	max = 2	max = 3
min = 0	1 <i>N, V</i> <i>NP, VP</i>		
min = 1		2 <i>N, M</i> <i>NP</i>	
min = 2			3 <i>N, V</i> <i>NP, VP</i>

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

Inner rules

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Preterminal rules

Figure 12: For some reason, they also consider unary rules, so in this case we also need to consider them. In particular, since  $V$  is a possible tag for “lead”, and there is a production  $VP \rightarrow V$ ,  $VP$  is also a valid parse for “lead”.

## CKY in action

lead	can	poison
0	1	2

		max = 1	max = 2	max = 3
min = 0	1 <i>N, V</i> <i>NP, VP</i>	4 <i>NP</i>		
min = 1		2 <i>N, M</i> <i>NP</i>		
min = 2			3 <i>N, V</i> <i>NP, VP</i>	

 $S \rightarrow NP VP$ 
 $VP \rightarrow M V$ 
 $VP \rightarrow V$ 
 $NP \rightarrow N$ 
 $NP \rightarrow N NP$ 
 $N \rightarrow can$ 
 $N \rightarrow lead$ 
 $N \rightarrow poison$ 
 $M \rightarrow can$ 
 $M \rightarrow must$ 
 $V \rightarrow poison$ 
 $V \rightarrow lead$ 

Inner rules

Preterminal rules

Figure 13: We now consider parsing “lead can”. This relies on combining the parses of “lead” and “can”. The only possible combination which has a valid production is  $NP \rightarrow NNP$ , so the only possible label is  $NP$ .



# CKY in action

	lead	can	poison	
0	1	2	3	

		max = 1	max = 2	max = 3
min = 0	1 $N, V$ $NP, VP$	4 $NP$		
min = 1		2 $N, M$ $NP$	5 $S, VP,$ $NP$	
min = 2			3 $N, V$ $NP, VP$	

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

Inner rules

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Preterminal rules

Figure 14: Similarly, for “can poison”, we consider the parses of “can” and “poison”. We can see there are 3 possible productions:  $S \rightarrow NPVP$ ,  $VP \rightarrow MV$  and  $NP \rightarrow NNP$ .

Now for “lead can poison”, we need to consider 2 types of parses: “lead” + “can poison” and “lead can” + “poison”.

# CKY in action

	lead	can	poison
0	1	2	3

max = 1      max = 2      max = 3

mid = 1

min = 0	1 <i>N, V</i> <i>NP, VP</i>	4 <i>NP</i>	6 <i>S, NP</i>
min = 1		2 <i>N, M</i> <i>NP</i>	5 <i>S, VP,</i> <i>NP</i>
min = 2			3 <i>N, V</i> <i>NP, VP</i>

*S* → *NP VP*

*VP* → *M V*  
*VP* → *V*

*NP* → *N*

*NP* → *N NP*

Inner rules

*N* → *can*  
*N* → *lead*  
*N* → *poison*

*M* → *can*  
*M* → *must*

*V* → *poison*  
*V* → *lead*

Preterminal rules

Figure 15: For “lead” + “can poison”, there are only 2 possible productions:  $S \rightarrow NPVP$  and  $NP \rightarrow NNP$ .

# CKY in action

	lead	can	poison
0	1	2	3

max = 1      max = 2      max = 3

mid = 2

min = 0	1 $N, V$ $NP, VP$	4 $NP$	6 $S, NP$ $S(!)$
min = 1		2 $N, M$ $NP$	5 $S, VP,$ $NP$
min = 2			3 $N, V$ $NP, VP$

$S \rightarrow NP VP$

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

Inner rules

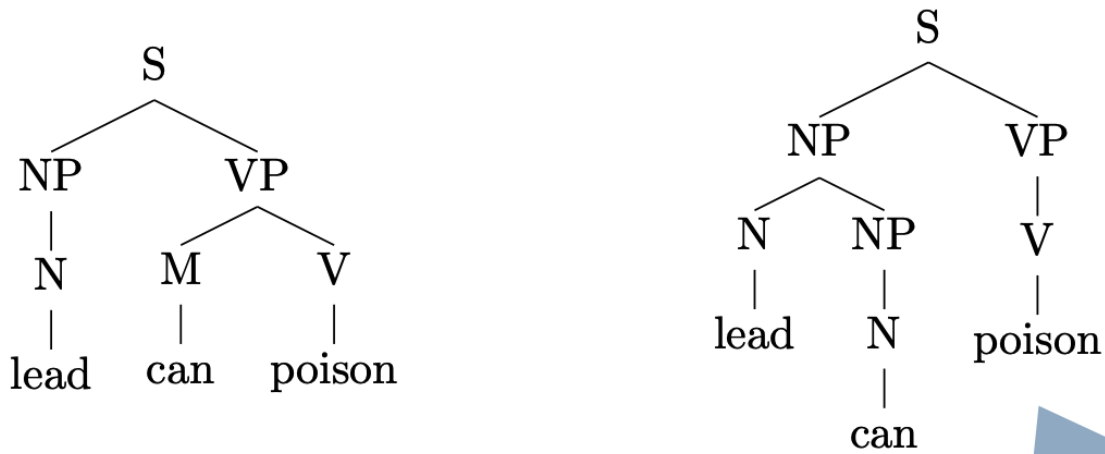
$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Preterminal rules

Figure 16: For “lead can” + “poison”, there is only 1 possible production:  $S \rightarrow NPVP$ .



No subject-verb agreement, and *poison* used as an intransitive verb

Figure 17: Hence, this sentence is ambiguous in the grammar, since it has 2 possible parse trees (but the second one is less likely - hint for the next section).

### 3 Statistical Parsing

#### 3.1 Probabilistic Context Free Grammars

- What are PCFGs?

- a natural **extension** of CFGs
- formally defined as **4-tuples**:
  1. **N**: set of **non-terminals**
  2. **Σ**: set of **terminal symbols**, **disjoint** from  $N$
  3. **R**: set of **productions**:

$$A \rightarrow \beta[p], \quad A \in N, \beta \in \Sigma, p \in [0, 1]$$

4. **S**: a **start symbol**

- here  $p$  is the **probability** of the production  $A \rightarrow \beta$ , where:

$$p = P(A \rightarrow \beta) = P(\beta \mid A)$$

such that:

$$\forall A \in N, \quad \sum_{\beta: A \rightarrow \beta \in R} P(A \rightarrow \beta) = 1$$

$S \rightarrow NP VP$	<b>1.0</b>	(NP A girl) (VP ate a sandwich)	$N \rightarrow girl$	<b>0.2</b>
			$N \rightarrow telescope$	<b>0.7</b>
$VP \rightarrow V$	<b>0.2</b>		$N \rightarrow sandwich$	<b>0.1</b>
$VP \rightarrow V NP$	<b>0.4</b>	(VP ate) (NP a sandwich)	$PN \rightarrow I$	<b>1.0</b>
$VP \rightarrow VP PP$	<b>0.4</b>	(VP saw a girl) (PP with ...)	$V \rightarrow saw$	<b>0.5</b>
			$V \rightarrow ate$	<b>0.5</b>
$NP \rightarrow NP PP$	<b>0.3</b>	(NP a girl) (PP with ....)	$P \rightarrow with$	<b>0.6</b>
$NP \rightarrow D N$	<b>0.5</b>	(D a) (N sandwich)	$P \rightarrow in$	<b>0.4</b>
$NP \rightarrow PN$	<b>0.2</b>		$D \rightarrow a$	<b>0.3</b>
			$D \rightarrow the$	<b>0.7</b>
$PP \rightarrow P NP$	<b>1.0</b>	(P with) (NP with a sandwich)		

- Why are PCFGs useful?

1. **Disambiguation:** allow us to compute the **probability of parse trees**, so we can pick the **most likely parse**
2. **Language Modelling:** allow us to compute the **probability of a sentence**

- How is the probability of a parse tree computed?

- consider a sentence  $S$  with parse tree  $T$
- then the probability of having  $S$  parsed with  $T$  is the product of all the productions used to expand **non-terminal** nodes. In particular, if  $n$  nodes are expanded using rules:

$$LHS_i \rightarrow RHS_i, \quad i \in [1, n]$$

then:

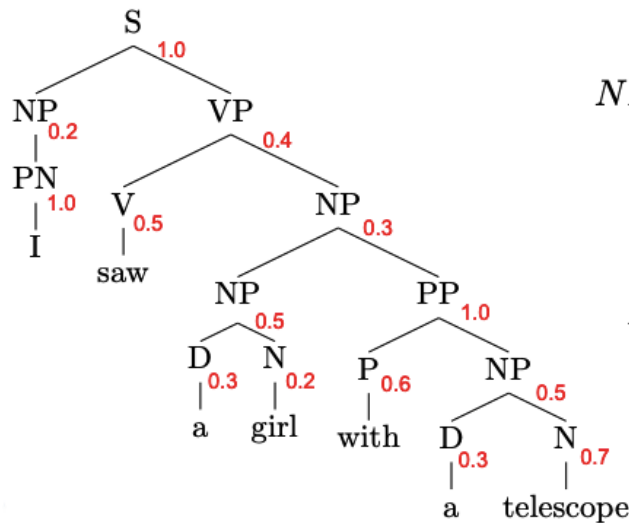
$$P(T, S) = \prod_{i=1}^n P(RHS_i \mid LHS_i)$$

- however, notice that:

$$P(T, S) = P(T)P(S \mid T) = P(T)$$

where we use the fact that  $P(S \mid T) = 1$  (that is,  $T$  always generates  $S$ , since it is its parse tree)

- thus, we can talk about the **probability of a parse tree**, independently of the sentence used to generate it (this makes sense, since the preterminal probabilities are not required to compute the probability  $P(T, S)$ )



$S \rightarrow NP VP$	1.0	$N \rightarrow girl$	0.2
$VP \rightarrow V$	0.2	$N \rightarrow telescope$	0.7
$VP \rightarrow V NP$	0.4	$N \rightarrow sandwich$	0.1
$VP \rightarrow VP PP$	0.4	$PN \rightarrow I$	1.0
$NP \rightarrow NP PP$	0.3	$V \rightarrow saw$	0.5
$NP \rightarrow D N$	0.5	$V \rightarrow ate$	0.5
$NP \rightarrow PN$	0.2	$P \rightarrow with$	0.6
$PP \rightarrow P NP$	1.0	$P \rightarrow in$	0.4
		$D \rightarrow a$	0.3
		$D \rightarrow the$	0.7

$$\begin{aligned}
 p(T) &= 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times 0.3 \times \\
 &\quad 0.5 \times 0.3 \times 0.2 \times 1.0 \times 0.6 \times 0.5 \times 0.3 \times 0.7 \\
 &= 2.26 \times 10^{-5}
 \end{aligned}$$

### 3.2 Issues with PCFGs

- What are the 2 key problems of PCFGs?
  1. they make an oftentimes invalid **independence assumption**
  2. no considerations for **lexical information**
- Why is the independence assumption poor?
  - PCFGs expand non-terminals **independently of context**
  - this is why we **multiply** expansion probabilities to compute the probability of a tree
  - this is an issue: for example, if an *NP* can be expanded in 2 ways, one expansion will be more likely than the other in a given context (i.e in a **subject**,  $NP \rightarrow PRP$  occurs 91% of the time, but the production  $NP \rightarrow PRP$  in general is much lower), but PCFGs don't capture these relationships
- Why is lexical information important?
  - PCFGs will be biased for more likely structures (i.e PP tends to attach to NP more often than to VP)
  - if lexical considerations were made, this probability could be more “refined” (i.e the preposition “into” has more affinity for the noun “sacks” - “Workers dumped sacks into a bin”; but the preposition “of” has more affinity for the verb “caught” - “Fishermen caught lots of fish”)

### 3.3 Most Likely Parse: A Probabilistic Distribution Over Parse Trees

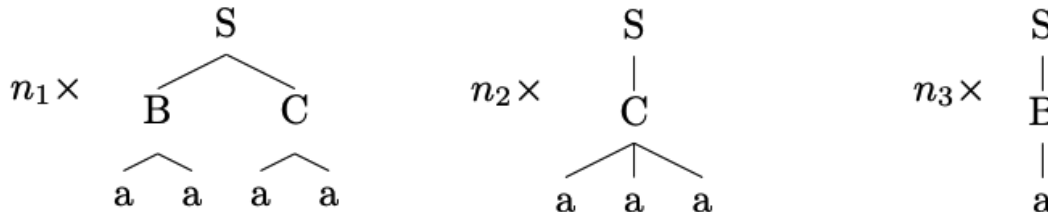
- How are production probabilities computed?

- the ML estimate is:

$$P(\alpha \mid X) = \frac{C(X \rightarrow \alpha)}{C(X)}$$

that is, the proportion of times in which the non-terminal  $X$  produced  $\alpha$

- these probabilities are typically computed using a **treebank**
- smoothing like **Good Turing** can be used (particularly helpful for preterminal productions)



Rule	Count	Prob. estimate
$S \rightarrow B C$	$n_1$	$n_1 / (n_1 + n_2 + n_3)$
$S \rightarrow C$	$n_2$	$n_2 / (n_1 + n_2 + n_3)$
$S \rightarrow B$	$n_3$	$n_3 / (n_1 + n_2 + n_3)$
$B \rightarrow a a$	$n_1$	$n_1 / (n_1 + n_3)$
$B \rightarrow a$	$n_3$	$n_3 / (n_1 + n_3)$
$C \rightarrow a a$	$n_1$	$n_1 / (n_1 + n_2)$
$C \rightarrow a a a$	$n_2$	$n_2 / (n_1 + n_2)$

- When is a PCFG consistent?

- when the **sum** of probabilities of **all** sentences in the grammar is 1
- this can happen with, for example, rules like  $S \rightarrow S[1]$ , which cause infinitely long strings

- What is a proper distribution over parse trees?

- when the **sum** of probabilities of **all** trees in the grammar is 1:

$$\sum_T P(T) = 1$$

- if we estimate probabilities using MLE, we are guaranteed a **proper** distribution

### 3.4 Probabilistic CYK

- What is the best parse according to a PCFG?

- PCFGs allow us to define a **best** parse, as the most likely parse tree:

$$\hat{T} = \underset{T \in G(x)}{\operatorname{argmax}} P(T)$$

where  $G(x)$  is the set of **all** derivations of a sentence  $x$

- finding all possible  $T$  is exponential in nature, so we need to use **probabilistic CYK**

- What is probabilistic CYK?

- extension of CYK, adapted to PCFGs in CNF
- when converting to CNF, the production probabilities need to be adapted
- for a sentence with  $n$  words, and a PCFG with  $V$  non-terminals, produces a  $n + 1 \times n + 1 \times V$  table
- entry  $(i, j, C)$  corresponds to the highest probability of  $\operatorname{span}(i, j)$  being a constituent of type  $C$

```
function PROBABILISTIC-CKY(words,grammar) returns most probable parse
                                     and its probability
for  $j \leftarrow$  from 1 to LENGTH(words) do
  for all {  $A \mid A \rightarrow \text{words}[j] \in \text{grammar}$  }
     $\text{table}[j-1, j, A] \leftarrow P(A \rightarrow \text{words}[j])$ 
  for  $i \leftarrow$  from  $j-2$  downto 0 do
    for  $k \leftarrow i+1$  to  $j-1$  do
      for all {  $A \mid A \rightarrow BC \in \text{grammar},$ 
                and  $\text{table}[i, k, B] > 0$  and  $\text{table}[k, j, C] > 0$  }
        if ( $\text{table}[i, j, A] < P(A \rightarrow BC) \times \text{table}[i, k, B] \times \text{table}[k, j, C]$ ) then
           $\text{table}[i, j, A] \leftarrow P(A \rightarrow BC) \times \text{table}[i, k, B] \times \text{table}[k, j, C]$ 
           $\text{back}[i, j, A] \leftarrow \{k, B, C\}$ 
  return BUILD_TREE( $\text{back}[1, \text{LENGTH}(\text{words}), S]$ ),  $\text{table}[1, \text{LENGTH}(\text{words}), S]$ 
```

Figure 18: Again, use backpointer to keep track of where the most likely probability comes from to reconstruct the tree.

```
for each  $w_i$  from left to right

  for each preterminal rule  $C \rightarrow w_i$ 

     $\text{chart}[i-1][i][C] = p(C \rightarrow w_i)$ 
```

Figure 19: For handling preterminal rules.



```

for each max from 2 to n

  for each min from max - 2 down to 0

    for each syntactic category C

      double best = undefined

      for each binary rule C -> C1 C2

        for each mid from min + 1 to max - 1

          double t1 = chart[min][mid][C1]

          double t2 = chart[mid][max][C2]

          double candidate = t1 * t2 * p(C -> C1 C2)

          if candidate > best then

            best = candidate

      chart[min][max][C] = best

```

Figure 20: For handling binary rules. Again, to handle the unary productions, a slight modification is required.

- **How can we deal with unary closure?**

- notice, if we have  $A \rightarrow B$  and  $B \rightarrow C$ , adding  $A \rightarrow C$  (by defining  $P(A \rightarrow C) = P(A \rightarrow B) \times P(B \rightarrow C)$ ), will break the consistency of the PCFG
- necessary, since a rule directly defined by  $A \rightarrow C$  might have a very low probability, but  $A \rightarrow B$  and  $B \rightarrow C$  might be very likely
- need to store the fact that  $A \rightarrow C$  is composite (i.e don't evaluate the product  $P(A \rightarrow B) \times P(B \rightarrow C)$ ), so that we can recover the parse tree
- rules like  $X \rightarrow X$  should have probability 1

- **Should PCFGs worry about infinite productions or loops?**

- no, since such productions will have infinitesimal probabilities (large products)
- since CYK selects largest probability, such situations won't be considered

- **How can probabilistic CYK be sped up?**

1. **Basic Pruning:** only store labels with high probabilities (i.e within a factor of  $N$  of the most likely label); only consider rules which lead to trees with non-zero probabilities
2. **Coarse-to-fine Pruning:** use simpler grammar to parse and precompute probabilities for each  $span(i, j)$ ; then consider labels with non-negligible probabilities for the full parse